PROBLEMS.

65. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Bought April 4, 1894, 250 yards of broadcloth at \$5.37½ per yard, less 12½ and 10% discount for cash payment. Sold September 5, 1894, at 15, 10, and 5% on *quoted price*, the cloth; and in settlement received a 90-day note which I had discounted at 5½%, October 19, 1894, by the First National Bank of Baltimore, Maryland. Reckoning 6% interest on the *money invested* in the cloth, what is the profit made?

66. Proposed by F. P. MATZ, M. Sc., Ph. D., Professor of Mathematics and Astronomy in Irving College, Mechanicsburg, Pennsylvania.

Brown adds $m=10\%$ of water to the pure wine he buys, and then sells the mixture at a price $n=10\%$ greater than the cost price of the pure wine. What is his rate per cent. of profit?

ALGEBRA.

Conducted by J. M. COLAW, Monterey, Va. All contributions to this department should be sent to him.

SOLUTIONS OF PROBLEMS.

62. Proposed by Professor C. E. WHITE, A. M., Trafalgar, Indiana.

Prove that every algebraic equation can be transformed into another equation of the same degree, but which wants its n^{th} term.

I. Solution by HENRY HEATON, M. Sc., County Surveyor, Atlantic, Iowa.

To illustrate, let $x^4+ax^3+bx^2+cx+d=0$ be any equation of the fourth degree. Put $x=y+p$; then the equation becomes

$$y^4+(4p+a)y^3+(6p^2+3ap+b)y^2+(4p^3+3ap^2$$
$$+2bp+c)y+p^4+ap^3+bp^2+cp+d=0.$$

Since we are at liberty to give p any value, we may give it the value that will make $4p+a=0$ or $-a/4$; then will the coefficient of y^3 disappear. It is also evident that we may give p such a value that any desired coefficient will disappear. It is also evident that to find the desired value of p by this method requires for the second term, the solution of an equation of the first degree; for the third term, the solution of an equation of the second degree, etc. It is further evident that this is true without regard to the degree of the original equation.

II. Solution by BENJ. F. YANNEY, A. M., Professor of Mathematics in Mount Union College, Alliance, Ohio.

If not already so, any equation of the n^{th} degree may be reduced to the form $x^n+Ax^{n-1}+Bx^{n-2}+\ldots\ldots+L=0$. Now, by putting for x, $x+a$, we obtain a new equation whose roots differ from the corresponding roots of the given equation by a, (and whose degree, therefore, is still the n^{th}) viz.:

$$x^n+(na+A)x^{n-1}+\left(\frac{n(n-1)}{\underline{|2}}a^2+(n-1)Aa+B\right)x^{n-2}$$

$$+\ldots\ldots+(a^n+Aa^{n-1}+Ba^{n-2}+\ldots\ldots+L=0.$$

As a is an arbitrary constant, it may be selected so that $(na+A)=0$, or

$$\left(\frac{n(n-1)}{\underline{|2}}a^2+(n-1)Aa+B\right)=0,$$

or any coefficient, except the first, $=0$. Hence, any term, except the first, may thus be removed.

III. Solution by O. W. ANTHONY, M. Sc., Professor of Mathematics and Astronomy in New Windsor College, New Windsor, Maryland.

Every algebraic equation may be written

$$X^k-\Sigma\alpha.X^{k-1}+\Sigma\alpha\beta.X^{k-2}\ldots\ldots=0.$$

The coefficient of the n^{th} term will be $\Sigma\alpha\beta\gamma\ldots\ldots$to $n-1$ factors. Now in place of X write $X+h$; then α, β, γ, etc., will be changed into $\alpha+h$, $\beta+h$, $\gamma+h$, etc. The coefficient of n^{th} term will then be $\pm\Sigma(\alpha+h)(\beta+h)(\gamma+h)\ldots\ldots$to $n-1$ terms. If we equate this to zero, we may consider it an equation of degree $n-1$ in h. This will give $n-1$ values of h. Therefore there are $n-1$ transformations which will make the n^{th} term vanish. Consider the first term, $n-1$; there are in that case no transformations.

Also solved by *PROF. E. W. MORRELL.*

63. Proposed by J. A. CALDERHEAD, A. B., Professor of Mathematics in Curry University, Pittsburg, Pennsylvania.

Given $x^2+x\sqrt{xy}=10$, and $y^2+y\sqrt{xy}=20$ to find x and y by quadratics.

I. Solution by E. L. BROWN, A. M., Professor of Mathematics, Capital University, Columbus, Ohio; HENRY HEATON, M. Sc., Atlantic, Iowa; and G. B. M. ZERR, A. M., Ph. D., Texarkana, Arkansas-Texas.

Factoring, we have $x^{\frac{3}{2}}(x^{\frac{1}{2}}+y^{\frac{1}{2}})=10$, $y^{\frac{3}{2}}(x^{\frac{1}{2}}+y^{\frac{1}{2}})=20$.

$$\therefore y^{\frac{3}{2}}/x^{\frac{3}{2}}=2,\ y^{\frac{3}{2}}=2x^{\frac{3}{2}}.\quad \therefore y=\sqrt[3]{4}x.$$